Computer Science II - Mid-term exam
Friday 08 March 2013, 10:00-13:00

This is a closed book exam. Show all your work.
Correct answers with insufficient or incorrect work will not get any credit.
Maximum possible score is 120. There are eight questions.

March 8, 2013

Signature: _____

Name: _____

1. (10 points) Let $a = 121$, $b = 0.327$, $c = 119$. Find $d = (a - b) - c$ and $e = (a - c) - c$ using three digit arithmetic with chopping off. What are the absolute and relative errors in each of these cases?

2. (10 points) Suppose $x_n = O(y_n)$. Prove or disprove: $y_n^{-1} = O(x_n^{-1})$.

3. (10 points) Find the rates of convergence of the following two sequences (k is some positive integer).

$$a_n = \frac{1}{n^k} \qquad b_n = 10^{-(2^n)} \tag{1}$$

4. (20 points) The roots of a certain quadratic are given by $r = -p + \sqrt{p^2 + q}$. Write down this method of calculating the root as an algorithm. What is the inherent error in a machine with unit roundoff error ε? When $p \gg q$, show that this algorithm is numerically unstable.

5. (20 points) Consider the function $f(x) = \text{sign}(x)\sqrt{|x|}$. Show that the Newton's method does not converge to the root for any initial guess x_0. Explain why it does not converge.

6. (20 points) Suppose f is a continuous function on $[a, b]$ and $f(a)f(b) < 0$. If we use $[a, b]$ as the initial bracket of the bisection method to find a root r in that interval, what is the minimum number of iterations needed to find the root with an absolute error less than ε?

7. (10 points) Write the output of the following Matlab/octave commands.

```
octave:1> m = [eye(3)  (11:13)' ;  (21:24) ]
octave:2> x = -1.5:0.5:1
octave:3> y = 1:3
octave:4> [aa bb] = meshgrid(x,y)
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8. (20 points) Consider a sequence (x_n, y_n) defined by $x_{n+1} = y_n$, $y_{n+1} = -x_n - y_n^3$. Write a matlab/octave function whose input will be a pair of real numbers (r, s) and a positive integer K and whose output will be two vectors X and Y of dimension $(K+1) \times 1$ each, consisting of iterates x_n and y_n, respectively, of this sequence for $n = 0, 1, \ldots, K$ with $x_0 = r$ and $y_0 = s$.